Transient Finite Element Simulation of Non-Linear Eddy Current Problems with Biot-Savart-Field of Voltage-Driven Coils

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A method is presented to carry out a transient simulation of eddy current problems with nonlinear materials. Coils are voltage-driven. The magnetic field due to currents in coils are considered by their Biot-Savart-fields. The magnetic vector potential is used in the finite element formulation. The time stepping method is based on implicit Euler. The arising nonlinear equation system is split into two parts, each part is solved separately by Newton's method. The inrush current of a benchmark problem is studied.

***Index Terms*—Biot-Savart-field, finite element method, inrush current, nonlinear eddy current problem, voltage-driven coil.**

I. Introduction

THE METHOD presented here facilitates a transient finite element simulation of nonlinear eddy current problems with the magnetic vector potential (MVP) involving voltage-driven coils. An early work for a voltage-driven coil and the finite element method (FEM) is [1]. Unlike here a method based on harmonic balance and current vector potential has been introduced in [2]. The Biot-Savart-field (BSF) caused by a current in a coil is calculated only once and then accordingly scaled to the course of time. The arising nonlinear equation system is split into two parts, Newton's method is applied to each part separately. The two parts are alternately solved until a stopping criterion is fulfilled. Induction effects in the windings are neglected (stranded coils). A solution for that can be found for linear materials and BSF in [3]. The paper presents the case of one coil in detail, the extension to several coils is straight forward and outlined at the end. A study of a numerical benchmark is presented.

II. Eddy Current Problem

A. Boundary Value Problem

The eddy current problem to be solved is sketched in Fig. 1. It consists of a conducting domain (iron) Ω_c and air Ω_0, i.e., $\Omega = \Omega_c \cup \Omega_0$ with the boundary $\Gamma = \Gamma_D \cup \Gamma_N$. The eddy current problem with the MVP $\boldsymbol{A}$ in the time domain reads as where $\boldsymbol{J}_0$ in (1) stands for a known source current density in

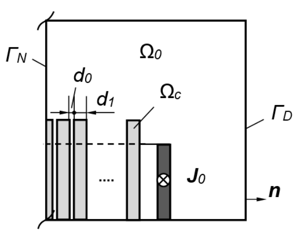

Fig. 1. One eighth of the boundary value problem, coil with laminated iron core.

$$\operatorname{curl} \frac{1}{\mu(\boldsymbol{A})} \operatorname{curl} \boldsymbol{A} + \sigma \frac{\partial}{\partial t} \boldsymbol{A} = \boldsymbol{J}_0 \quad \text{in } \Omega \subset \mathbb{R}^3, \tag{1}$$

$$\boldsymbol{A} \times \boldsymbol{n} = \boldsymbol{\alpha} \quad \text{on } \Gamma_D, \tag{2}$$

$$\frac{1}{\mu(\boldsymbol{A})} \operatorname{curl} \boldsymbol{A} \times \boldsymbol{n} = \boldsymbol{K} \quad \text{on } \Gamma_N, \tag{3}$$

$\Omega_s \subset \Omega_0$, $\boldsymbol{\alpha}$ in (2) represents a magnetic flux and $\boldsymbol{K}$ in (3) describes a surface current density. The material parameters are the magnetic permeability $\mu(\boldsymbol{A})$ and the electric conductivity σ, respectively.

B. Weak Form

Equations (1) to (3) lead to the following weak form for the FEM. Find $\boldsymbol{A}_h \in \mathcal{V}_\alpha := \{\boldsymbol{A}_h \in \mathcal{V}_h : \boldsymbol{A}_h \times \boldsymbol{n} = \boldsymbol{\alpha}_h \text{ on } \Gamma\}$, such that

$$\int_\Omega \frac{1}{\mu(\boldsymbol{A}_h)} \operatorname{curl} \boldsymbol{A}_h \cdot \operatorname{curl} \boldsymbol{v}_h \, d\Omega + \frac{\partial}{\partial t} \int_\Omega \sigma \boldsymbol{A}_h \cdot \boldsymbol{v}_h \, d\Omega = \int_{\Omega_s} \boldsymbol{J}_0 \cdot \boldsymbol{v}_h \, d\Omega + \int_{\Gamma_N} \boldsymbol{K} \cdot \boldsymbol{v}_h \, d\Gamma \tag{4}$$

for all $\boldsymbol{v}_h \in \mathcal{V}_0 := \{\boldsymbol{v}_h \in \mathcal{V}_h : \boldsymbol{v}_h \times \boldsymbol{n} = \boldsymbol{0} \text{ on } \Gamma_D\}$, where $\mathcal{V}_h$ is a finite element subspace of $H(\operatorname{curl}, \Omega)$.
For the sake of clarity $\boldsymbol{K}$ has been omitted in the following considerations. Regularization is applied in Ω_0 [4]. Hexahedral edge elements [5] were used to facilitate the modeling of laminates.

C. Voltage-driven Coil and Biot-Savart-Field

The network equation

$$u_0(t) = i(t) R_s - \frac{d\phi(t)}{dt} \tag{5}$$

with voltage u_0 of the voltage source, the current i, the series resistor R_s and the magnetic flux ϕ has to be solved together with (4) simultaneously. Rearranging of the linear form in (4) yields

$$\int_{\Omega_s} \boldsymbol{J}_0 \cdot \boldsymbol{v}_h \, d\Omega = i(t) \int_{\Omega_s} \boldsymbol{\tau} \cdot \boldsymbol{v}_h \, d\Omega = \int_\Omega \operatorname{curl} \boldsymbol{H}_{BS} \cdot \boldsymbol{v}_h \, d\Omega = i(t) \int_\Omega \operatorname{curl} \boldsymbol{h}_0 \cdot \boldsymbol{v}_h \, d\Omega \tag{6}$$

with the turn density $\boldsymbol{\tau}$ and the BSF

$$\boldsymbol{H}_{BS} = \frac{1}{4\pi} \int_{\Omega_s} \frac{\boldsymbol{J}_0 \times (\boldsymbol{r}_F - \boldsymbol{r}_S)}{|\boldsymbol{r}_F - \boldsymbol{r}_S|^3} \, d\Omega, \tag{7}$$

where $\boldsymbol{r}_F$ and $\boldsymbol{r}_S$ are the field and the source point. Hence, $\boldsymbol{h}_0$ is the BSF of a unit current and note, that thanks to $\boldsymbol{\tau} = \operatorname{curl} \boldsymbol{h}_0$ the volume of the coil need not to be modeled by finite elements. At the same time rearranging of the second term on the right hand side of (5) leads to

$$-\frac{d\phi(t)}{dt} = \int_{Sw} \boldsymbol{E} \cdot d\boldsymbol{s} = \int_{S} \boldsymbol{E} \cdot \frac{w \boldsymbol{e}_F}{F} \cdot F \boldsymbol{e}_F \cdot d\boldsymbol{s} =$$

$$\int_{S} \boldsymbol{E} \cdot \boldsymbol{\tau} \cdot F \boldsymbol{e}_F \cdot d\boldsymbol{s} = \int_{\Omega} \boldsymbol{E} \cdot \boldsymbol{\tau} \cdot d\Omega = -\int_{\Omega} \frac{\partial \boldsymbol{A}}{\partial t} \cdot \boldsymbol{\tau} \, d\Omega \tag{8}$$

with F, $\boldsymbol{E}$, S, w and $\boldsymbol{e}_F$, respectively, the cross-section of the coil, the electric field strength, the average path length of a winding, the number of turns and the unit vector of the surface F, respectively.

D. Time Stepping

Evaluation of (4) and considering (6) on the one hand and (5) with (8) on the other results in the nonlinear ordinary differential equation system

$$\begin{aligned} \boldsymbol{S}(\boldsymbol{a})\boldsymbol{a}(t) + \boldsymbol{M}\frac{d}{dt}\boldsymbol{a}(t) - \boldsymbol{b}i(t) &= \boldsymbol{0} && (9) \\ -\boldsymbol{b}^T \frac{d}{dt}\boldsymbol{a}(t) + i(t)R_s &= u_0(t) && (10) \end{aligned}$$

In (9) and (10), $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{S}$ and $\boldsymbol{M}$, respectively, are the unknown vector of the MVP, the vector according to $\boldsymbol{h}_0$ and (6), the stiffness matrix and the mass matrix, respectively. Considering implicit Euler for time stepping provides the system of nonlinear equations

$$\begin{aligned} \boldsymbol{S}(\boldsymbol{a}^{k+1})\boldsymbol{a}^{k+1} + \frac{1}{\Delta t}\boldsymbol{M}\boldsymbol{a}^{k+1} &= \frac{1}{\Delta t}\boldsymbol{M}\boldsymbol{a}^{k} + \boldsymbol{b}i^k && (11) \\ -\boldsymbol{b}^T\boldsymbol{a}^{k+1} + \Delta t i^{k+1} R_s &= \Delta t u_0^{k+1} - \boldsymbol{b}^T\boldsymbol{a}^k && (12) \end{aligned}$$

to be solved, where Δt is the time step and k the index for the time instant $t_k = k\Delta t$. The system of (11) and (12) is symmetric.

E. Newton's method

The system to be solved is split into two parts according to (11) and (12), i.e. the unknown vector $\boldsymbol{a}$ and the current i. Let $\boldsymbol{F}(\boldsymbol{a}^{k+1}) = \boldsymbol{0}$ and $G(i^{k+1}) = 0$ be the explicit representations of (11) and (12). Applying Newton's method to (11) and (12) yields:

$$\begin{aligned} \boldsymbol{a}_{l+1}^{k+1} &= \boldsymbol{a}_l^{k+1} - \alpha \boldsymbol{J}_F^{-1}(\boldsymbol{a}_l^{k+1})\boldsymbol{F}(\boldsymbol{a}_l^{k+1}) && (13) \\ i_{l+1}^{k+1} &= i_l^{k+1} - J_G^{-1}(i_l^{k+1})G(i_l^{k+1}) && (14) \end{aligned}$$

Index l denotes the nonlinear iterations. Newton's method to solve (13) is supplemented by a line search using a corresponding functional of (4). The unknown parameter α indicates the line search. The Jacobian matrices are:

$$\begin{aligned} \boldsymbol{J}_F(\boldsymbol{a}^{k+1}) &= \frac{d}{d\boldsymbol{a}^{k+1}}\boldsymbol{F}(\boldsymbol{a}^{k+1}) && (15) \\ J_G(i^{k+1}) &= \frac{d}{di^{k+1}}G(i^{k+1}) && (16) \end{aligned}$$

To get (14)

$$\boldsymbol{J}_F(\boldsymbol{a}^{k+1})\frac{d\boldsymbol{a}^{k+1}}{di^{k+1}} = \boldsymbol{b} \tag{17}$$

has to be solved first.
While $\boldsymbol{a}^{k+1}$ of (13) is solved iteratively the current i^k is constant and known and vice versa. Stopping criteria are both a minimal change in $\boldsymbol{a}^{k+1}$ and i^{k+1} and a maximal number of iterations l.

The extension to an arbitrary number n of coils is straightforward. To this end (5) is replaced by the corresponding n network equations, (6) and (8) are written for each coil resulting in n different vectors $\mathbf{b}_i$, $i = 1, ..., n$.

III. Numerical Example

The 3D numerical benchmark consists of a laminated iron core (isotopic: M400-50A, 183 laminates) inserted in a cylindrical coil as indicated in Fig. 1. A handmade hexahedral mesh was created to simplify the modeling of the laminates. The Biot-Savart field was used to avoid the modeling of the cylindrical coil. The ohmic resistor R_s comprises the internal resistor of the coil and a series resistor. A study of inrush currents are shown in Fig. 2. Although the currents do not reflect the nonlinearity, the magnetic flux in the outer most laminate is clearly nonlinear (not shown here).

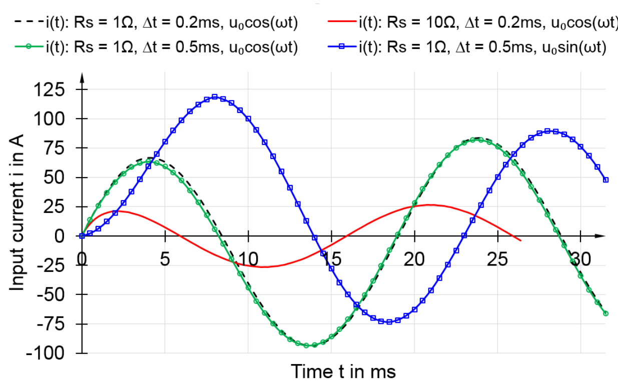

Fig. 2. Study of inrush currents for different R_s, Δt and $u_0(t)$.

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